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SMALL EXPENDABLE DEPLOYER SYSTEM MEASUREMENT ANALYSIS

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ABSTRACT

The first on-orbit experiment of the Small Expendable Deployer System (SEDS) for tethered satellites will collect telemetry data for tether length, rate of deployment, and tether tension. The post-flight analysis will use this data to reconstruct the deployment history and determine dynamic characteristics such as tether shape and payload position. Linearized observability analysis has determined that these measurements are adequate to define states for a two-mass tether model, and two state estimators have been written.

ACKNOWLEDGEMENTS

The summer program participant would like to thank Program Development at Marshall Space Flight Center for a most enjoyable summer of research. The SEDS analysis provided a "real-life" problem that has been educational to the investigator, and will be discussed in her estimation course. In particular, I would like to thank Chris Rupp and Jim Harrison for all their help in defining the problem and their advice in addressing it.

The investigator would also like to acknowledge the excellent summer program that Mike Freeman and his staff have created. It was a pleasure to be a part of this program.

INTRODUCTION

The first flight of the tethered Small Expendable Deployer System (SEDS) will be launched from a Delta II. A 14 kg payload will be deployed downward to a tether length of 2 km, where it will librate to vertical and the tether will be cut.

The data system will record the number of reel-turns as the tether is deployed, and tether tension. These measurements will be downlinked to a ground station so that deployment can be monitored, and the data will be stored for post-flight analysis.

The telemetry data will provide measurements of tether length, rate of deployment and tether tension. To determine if these measurements are adequate for reconstruction of the deployment dynamics, system observability calculations have been made (based on a constantly-updated linear model), and two state estimators have been developed.

OBJECTIVES

The objectives of the summer faculty research were to:

1. Determine if the turns-count and tension measurements are adequate to reconstruct the deployment dynamics.
2. Determine if the measurement sampling rate is adequate.

STATE MODEL

Several computer simulations of tether deployment dynamics are available, ranging from planar simple-pendulum representations to three-dimensional partial-differential-equation models. The summer investigator chose Energy Science Laboratories (ESL) BEADSIM model to provide the tether dynamics state equations, since it is relatively simple and yet still produces results that are comparable to more complex models. BEADSIM is a lumped mass model, in which masses or "beads" are added as the tether becomes longer. No out-of-orbit-plane motion is modelled, and the external forces on each bead are the gravity gradient, aerodynamic drag, and Coriolis and centripetal accelerations. The equations are written using a Cartesian coordinate frame with an origin at the center of mass and moving at orbit speed (Fig. 1).

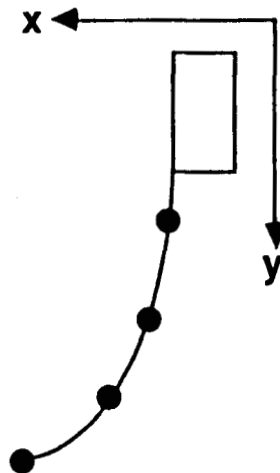


FIG. 1. BEADSIM Tether Model.

Each bead's motion is governed by a second-order differential equation, with a uniform tether tension providing the coupling between beads. Four states represent the motion of each bead: the x and y positions, and the x and y velocities. Deployment

characteristics such as tether shape, vibration amplitudes, and payload position and velocity are determined as the states change in time.

Simulations like BEADSIM are dependent on initial conditions for the states; any changes in the initial conditions will produce a different deployment trajectory (Fig. 2). The SEDS measurements would not be necessary for post-flight analysis if the initial positions and velocities of the deployer and payload were known exactly, and if parameters such as the aerodynamic drag coefficient were accurate. However, variations in the initial conditions greatly effect tether deployment time, for example, so that an estimate of initial conditions is not adequate to determine deployment characteristics.

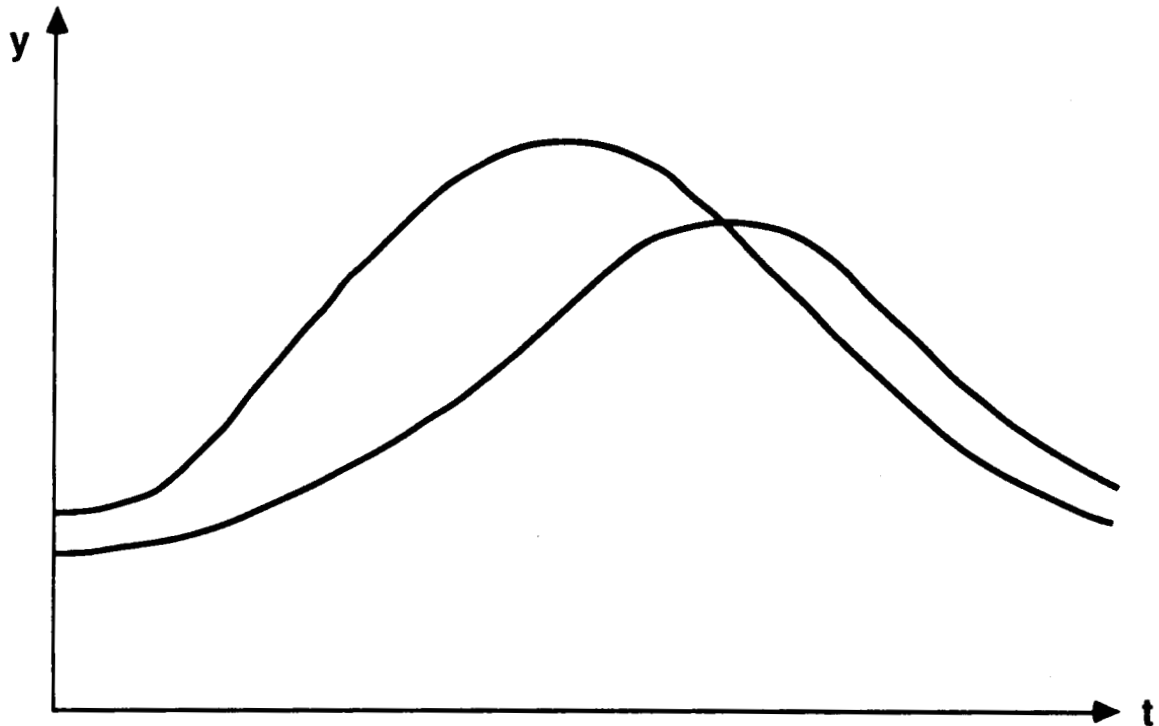


FIG. 2. Changes in the Initial Conditions Produce Different Deployment Trajectories.

The SEDS measurements consist of tether length l , length rate \dot{l} , tension T , and the Delta second stage

position and velocity. These measurements are nonlinear functions of the states, represented by the following measurement equations

$$l = G(y) \quad (1)$$

where l represents the vector of measurements at a given time, and y the corresponding states. The BEADSIM state equations are also nonlinear, and may be written as follows

$$\dot{y} = F(y) \quad (2)$$

The measurement and state equations will be used to determine system observability and to develop the state estimators.

SYSTEM OBSERVABILITY ANALYSIS

Given state and measurement equations like those in (1) and (2), a system is totally observable if all states can be determined from the measurements. For linear systems, a full-rank observability matrix ensures system observability. Since our system is nonlinear, a linear approximation based on the first term of a Taylor series will be used, so that the measurement equations (1) become

$$l = Hy \quad (3)$$

and the state equations become

$$\dot{y} = Ay \quad (4)$$

where H and A are the following matrices

$$H = \frac{\partial G}{\partial y} \bigg|_{y_{\text{current}}} \quad (5)$$

$$A = \frac{\partial F}{\partial y} \bigg|_{y_{\text{current}}} \quad (6)$$

Closed-form expressions for the elements in these matrices were derived from the functions F and G , so that errors from numerical differentiation were avoided. Since these matrices are evaluated at the current values for the states y , they will change as the states change. Hence system observability based on these matrices must be checked at each time step.

The observability matrix is defined as

$$O = \begin{bmatrix} H \\ HA \\ \vdots \\ HA^{n-1} \end{bmatrix} \quad (7)$$

where n is the number of states to be estimated. A is an 8×8 matrix for eight states corresponding to two beads, and H is a 7×8 matrix for the seven measurements l , \dot{l} , tension T , and the x , y position and velocity of the Delta. The observability matrix O is 56×8 . The rank is the number of independent rows in O , and is calculated using the singular value decomposition. The number of nonzero singular values at each time step was always eight, indicating that the linearized system for two beads is totally observable. Hence the positions and velocities of the tether deployer and payload may be determined by these measurements.

The rank of the observability matrix was also calculated without the tension measurement, and the system was still totally observable. This analysis indicates that tension measurements are not necessary to reconstruct the tether dynamics using two beads.

STATE ESTIMATORS

The measurements are not the same variables as the states, so that deployment characteristics such as tether shape cannot be directly determined from the measurements. Furthermore, the measurement process adds noise. A state estimator will use the measurements as input, filter the measurement noise, and then output the values of the states. Two estimators have been written: a batch estimator that finds the best initial conditions to minimize measurement errors, and a Kalman estimator that uses the current measurements to estimate the states. Both estimators process simulated measurements generated by BEADSIM to which white noise was added. The standard deviations of the noise are listed in Table 1.

TABLE 1 Standard Deviations for Measurement Noise

Measurement		σ
length	l	0.01 m
length rate	\dot{l}	0.001 m/s
tension	T	0.01 N
Delta	$\begin{cases} x_d \\ y_d \end{cases}$	1.0 m
position	$\begin{cases} \dot{x}_d \\ \dot{y}_d \end{cases}$	1.0 m
Delta	$\begin{cases} \ddot{x}_d \\ \ddot{y}_d \end{cases}$	0.001 m/s
velocity	$\begin{cases} \ddot{x}_d \\ \ddot{y}_d \end{cases}$	0.001 m/s

Least-Squares Batch Estimator

The least-squares batch estimator processes a batch of measurements to estimate initial conditions. The user inputs approximate initial conditions y_0 for the states, and the estimator integrates forward over the number of time steps for which measurements are available. The integrated values for the states at each time step are used to calculate measurement values \hat{l} for tether length, length rate, etc., which are then compared to the actual measurements l . The error vector $\{l - \hat{l}\}$ between the actual and calculated measurements is multiplied by a gain matrix that minimizes the errors in a least-squares sense, and the result is added to the

guess for the initial conditions y_0 . This procedure is based on Gaussian differential correction, and is repeated until the measurement errors $\{1-\hat{1}\}$ are small enough. The current iterate for the initial conditions is assumed to be correct, and the last set of integrated state values represent the true states.

Since BEADSIM adds more states as the tether is deployed, the measurements can be divided into batches that change when a bead is added. When enough tether has been deployed to add a third bead, the current batch of measurements is processed to determine initial conditions for the first two masses. The next batch of measurements is used to determine initial conditions for the third bead, and the process continues until all beads have been added.

Estimators for dynamic systems use the linearized system's transition matrix to define the relationship between initial conditions and later state values. The state transition matrix $\Phi(t, t_0)$ is the transformation that takes the initial state y_0 into the later state y

$$y(t) = \Phi(t, t_0)y(t_0) \quad (8)$$

For linear systems, the state transition matrix obeys the same differential equation as the states themselves

$$\dot{\Phi}(t) = A\Phi \quad (9)$$

and can be integrated forward in time along with the state equations. For nonlinear systems, however, equation (9) is a linearized approximation valid only in a neighborhood of the current state value. As the integration progresses and the current state gets further from the initial conditions, the approximation becomes inaccurate and the state transition matrix does not produce the same state values as the integrated state equations. This inaccuracy reduces the ability of the least-squares batch estimator to process measurements far from the initial conditions. Although the batch estimator appeared to estimate the correct initial conditions for two-bead tether simulations, the

principal investigator feels that a Kalman state estimator is more accurate.

Extended Kalman Estimator

Kalman filters are based on linear system theory, but can be used with an integrator to extend the algorithm to nonlinear processes. Unlike the batch estimator, measurements are processed one-at-a-time, so the current set of measurements are used to estimate the current state.

As an example, consider a Kalman estimator that processes a measurement l and estimates a state y . An integrator predicts an estimate of the state y_1 at time t_1 , and that estimate is used to calculate a value for the measurement \hat{l} . The actual measurement l is compared to \hat{l} , and the error $\{l - \hat{l}\}$ is multiplied by a gain G . $G(l - \hat{l})$ is the correction that is added to y_1 to give a better estimate of the state at time t_1 . The integrator goes forward another time step using the corrected state value at time t_1 , and a new correction is calculated from the next measurement.

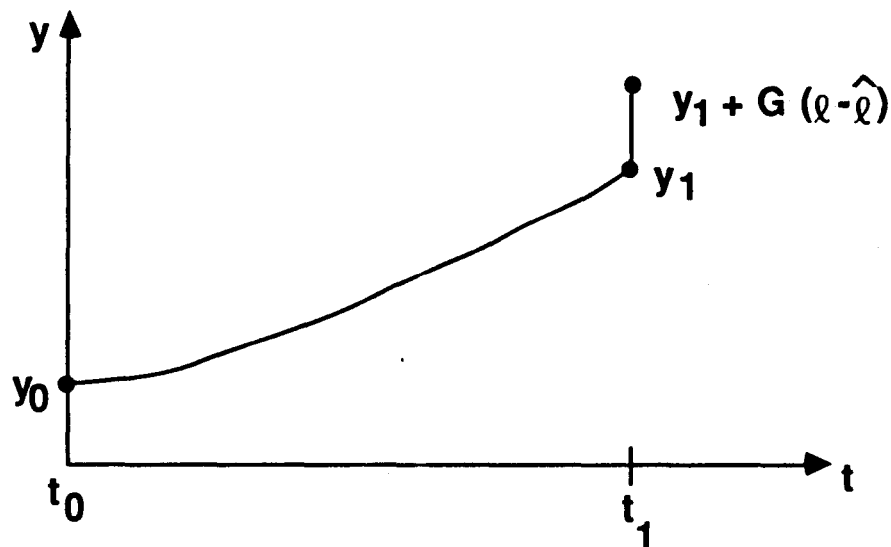


Fig. 3. Kalman Estimator Process.

The gains G are calculated by a minimum variance estimation algorithm based on assumed measurement noise statistics. Process noise statistics may also be included, so that the measurement noise and process noise become adjustments to tune the filter.

The extended Kalman filter uses the full nonlinear state equations to predict the next state, but still relies on the linearized state model for the state transition matrix, and a linearized measurement equation. Unlike the batch estimator, the Kalman filter only needs the state transition matrix that takes the current state to the next state

$$y(t_2) = \Phi(t_2, t_1)y(t_1) \quad (10)$$

so that the linearizing approximations that made the batch estimator inaccurate do not occur. As a check, the estimator can multiply the current state $y(t_1)$ by the state transition matrix and compare the result with that produced by integration of the nonlinear equations. Throughout the deployment, the two-bead state values agree to at least four decimal digits using a one-second time step, indicating that this time step is small enough to accommodate any linear approximations in the Kalman filter.

Several algorithms are available for coding the Kalman filter. The traditional filter algorithm produced overflows, so the U-D square-root factorization algorithm was programmed. This coding is more stable according to the published literature, but although it did not overflow it could not accurately estimate the tether angle during deployment. It was also extremely sensitive to assumed noise levels that tune the filter. Since the U-D algorithm could not produce satisfactory results, the traditional algorithm was modified to include process noise, and with this modification it successfully estimated the (two-bead) tether dynamics throughout deployment. The maximum error between the "true" states from BEADSIM and the estimated states was 10%, and occurred in the vertical position of the endmass. This error results from using only two-beads

in the estimator, as compared with the 21 beads used the generate the "true" states. A slight reduction in error was achieved by adjusting the masses of the two beads as the tether deployed, but a multibead capability must be added to the estimator to further reduce errors.

CONCLUSIONS AND RECOMMENDATIONS

The system observability analysis has indicated that tether length, length rate, and Delta position and velocity measurements are adequate to estimate positions and velocities of a two-bead tether model. The Kalman filter produces estimates within 5% of the "true" states for the first three-quarters of deployment, with a maximum of 10% error when fully deployed. Both the observability analysis and the estimators indicate that tension measurements are not necessary; noise levels and error bias associated with tension measurements may actually degrade the estimation process. The data sampling rates proposed at this time appear to be adequate to produce measurements at the one-second time interval needed by the Kalman estimator.

Recommendations for improvement include the addition of software to add more beads in the estimation process. The partial derivatives for the A and H matrices of equations (5) and (6) must be derived for multiple beads, and mechanisms developed for adding columns and rows to the gain and covariance matrices when beads are added. This improvement will reduce errors as well as add the capability to estimate flexible tether shape and vibrations.

The measurement noise added for simulation purposes was not as "white" as desired, due to correlation in the Turbo-Pascal random number generator. Noise models that are statistically more correct should be used, and accurate noise models for the tether length and length rate measurements should be developed. This would entail the development of a tether-reel dynamics model that would translate the turns-count data into length and length-rate measurements. The model should reflect the periods and biases associated with changes in the winding directions.

An alternate system model using polar coordinates l and θ rather than the Cartesian coordinates of BEADSIM may produce a more accurate estimator. The tether length and length rate measurements will then be linear functions of the states, which may reduce current errors due to linearization.

REFERENCES

Bierman, Gerald J., "Measurement Updating Using the U-D Factorization," Proceedings of the 1975 IEEE Conference on Decision and Control, pp. 337-346.

Bierman, Gerald J. and Thornton, Catherine L., "Numerical Comparison of Kalman Filter Algorithms: Orbit Determination Case Study," Automatica, Vol. 13, 1977, pp. 23-35.

Carroll, Joseph A., "Tether Applications in Space Transportation," Acta Astronautica, Vol. 4, 1986.

Carroll, Joseph A., "SEDS Data System Functional Requirements," report to MSFC Program Development, May 1988.

Carroll, Joseph A., "The Small Expendable Deployment System (SEDS)," presented at the PSN/NASA/ESA Second International Conference on Tethers in Space, Venice, Italy, October 1987.

Carroll, Joseph A. and Alexander, Charles M., "SEDS, The Small Expendable-tether Deployment System," Final Report NASA SBIR Phase II, NAS8-35256, December 1987.

Control Dynamics Company, "Evaluation of Tether Dynamics and Control System Interaction," NASA Final Report, NAS8-34667, June 1983.

Junkins, John L., An Introduction to Optimal Estimation of Dynamical Systems, Sijthoff & Noordhoff International Publishers, Alphen aan den Rijn, The Netherlands, 1978.

Kohler, P., Maag, W., and Wehrli, R., "Dynamics of a System of Two Satellites Connected by a Deployable and Extensible Tether of Finite Mass," NASA Final Report, October 1978.

Press, William H., Flannery, Brian P., Teukolsky, Saul A., and Vetterling, William T., Numerical Recipes: The Art of Scientific Computing, Cambridge University Press, 1986.

Wolovich, W.A., Linear Multivariable Systems, Springer-Verlag, New York, 1974.